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Fermionic Currents in Cosmic Strings

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Currents in cosmic strings built out of fermionic modes coupled to the vortex-forming Higgs field are reviewed. Massive modes are also exhibited which illuminate the structure of bosonic currents as the timelike charge is increased.

INTRODUCTION

Extensions of the simplest models of cosmic strings [1, 2], such as the one envisaged by Witten [3], involve extra degrees of freedom which are coupled to the vortex-forming Higgs field. This is the source of currents, bound to the core of the vortices, that may play a fundamental role in the dynamics of the defects. The superconducting charge carriers may be bosons if, for instance, a charged Higgs field acquires an expectation value in the core of the string different from the one it has outside. The carriers can also be fermions if these are trapped in bound modes (among which are the celebrated zero ones) along the vortex. Grand unified models like $SO(10)$ provide us with strings having right-handed neutrino zero modes [4]. Generically, a string can build up a random current (neutral in the latter case) similar to that in the bosonic case, resulting from the string self-interactions (like intersections and intercommutings). As the string self-intersects or intercommutes there is a finite probability that the Fermi levels will be excited. This results in a current flow smaller in general than the one we would get with the aid of an external magnetic field, but still sizable for grand unified theories. Either way the observational perspectives are even more interesting than those for nonconducting strings:

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they could indirectly be observed as synchrotron sources and as high-energy cosmic-ray sources [5], or directly be means of time-varying gravitational lenses [6].

Equally important is to study the possible fundamental fields that may constitute the charge carriers. In this respect, there have been proposed in the literature not only vector fields [7] and effective scalar fields, but also ordinary quark and lepton currents. In the latter case the strings will have important baryon-number-violating interactions with magnetic fields and are expected to be responsible for the yet-unexplained primordial baryogenesis [see, e.g., ref. 8]. Recently also supersymmetric partners, like squarks and sfermions, have been proposed as candidates for the conducting properties of the strings [9]; in fact, many of the grand unified theories at energies well above the electroweak scale predict the existence of this kind of stable string, so it is of uttermost importance that it be well understood.

Investigation of fermionic currents [10] was originally made through an explicit derivation of zero modes, i.e., solutions of a modified Dirac equation in the field of the vortex having vanishing energy, from which one can build arbitrary spacelike currents simply by filling up the various allowed states à la Dirac. It was then realized by Hindmarsh [11] that there are also solutions of the timelike kind leading to ordinary conductivity as one fills them.

In this work, we shall recap most of the results concerning these two kinds of modes to show the similarities with bosonic currents, in particular in view of the existence in both cases of a clearly defined *phase frequency threshold*, namely, the existence of a state of the vortex where the macroscopic charge diverges and the otherwise generically huge tension tends to vanish.

Thus, the bound-state solutions of the Dirac equation in a cosmic string vortex will be investigated in the framework of specific microscopic models; the present report represents some work currently in progress. We will concentrate on a reasonably generic kind of model, supposed to reproduce most of the characteristic features expected of a more realistic (and complicated) theory, and study the existence of timelike modes. After the setup of the relevant equations one needs to propose an adequate ansatz for the intervening fields, in the case under consideration, the string-forming Higgs, the gauge vector field responsible for the confinement of the string configuration, and the fermionic field, responsible for the existence of the current. While the approximate asymptotic behavior in particular restricted cases for these fields can always be analytically obtained, we are presently preparing the ground for the complete solution of the coupled system of dynamical evolution equations numerically. The study of the bound states, including zero energy modes along the vortex, will give us a unique insight into the current generation mechanisms and of the overall physics regarding the stability of the different string and loop configurations.

1. ZERO MODES AND THE WITTEN CONSTRUCTION

The basic idea in fermionic superconductivity in cosmic strings lies in the observation by Jackiw and Rossi [10] that there exist n zero-mode solutions of the "Dirac" equations in the background of an n-vortex of the Nielsen–Olesen kind [12], i.e., a solution of the field equations for a local $U(1)$ symmetry-breaking model involving a Higgs field Φ and an associated vector B_μ, having the form

$$\Phi = \varphi(r) \exp(in\theta) \tag{1}$$

$$B_\mu = B(r)\delta_{\mu\theta} \tag{2}$$

where r and θ are cylindrical coordinates, the string under consideration being considered along the z axis. The functions $\varphi(r)$ and $B(r)$, referred to hereafter as string profile functions, in principle solve the nonlinear field equations [13], and in particular one may impose the vanishing of the Higgs expectation value at the origin, $\varphi(0) = 0$, as a necessary topological requirement. The fermionic model comes into play when one considers a specific chiral fermion acquiring its mass from a Yukawa coupling with the Higgs field.

It follows from the mass generation mechanism for the fermions that they might be expected to be massless in the string's core where the Higgs field vanishes. This turns out to be a correct statement, in the sense that one can find solutions for a chiral spinor field $\Psi \propto \exp[-i(kz - \omega t)]$, where $\omega^2 = k^2$, the so-called zero modes, of vanishing energy. The crucial point in this analysis is that one considers chiral fermions. As a result, left-handed and right-handed particles (usually called left and right movers in a cosmic string context) behave differently under the influence of external fields, as exemplified on Fig. 1.

In the case where the fermions are coupled with the electromagnetic field, the resulting current J will satisfy, when an external electric field $\mathbf{E}$ is applied along the string ($E \equiv E_z$) [3]

$$\frac{dJ}{dt} = \frac{q^2}{\pi} E \tag{3}$$

where q is the electric charge (coupling constant) of the fermion under consideration and t the elapsed time. This is what is usually meant by *super-*

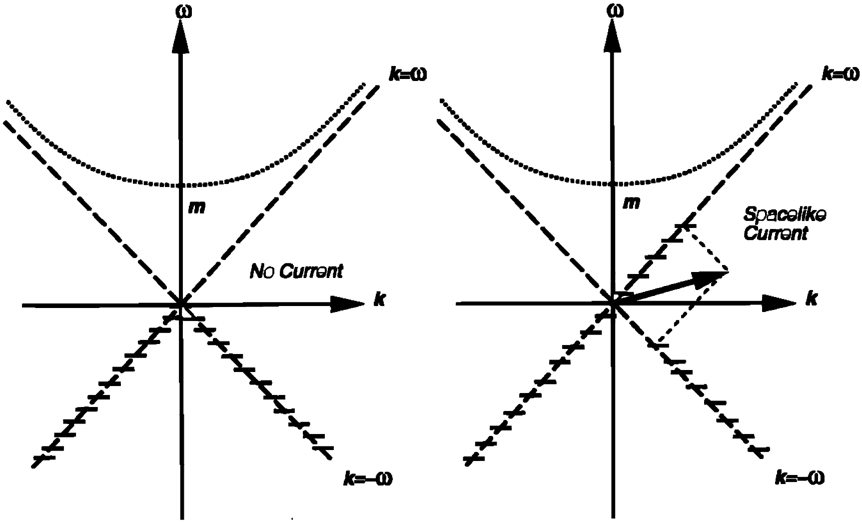

Fig. 1. Phase diagram for the zero modes. Left: The ground state when no external field is applied: the negative energy states are all filled, so the left and right movers both have vanishing Fermi energy. Right: An external electric field has been applied (or some other mechanism has been invoked), thereby shifting the Fermi levels in different directions; a spacelike current results. Also shown on this figure is the parabola $\omega^2 = k^2 + m^2$, with m the fermion mass asymptotically (i.e., where the Higgs field has its usual vacuum expectation value, far from the string's core). It should be clear that when the current corresponds to Fermi levels above the point $\omega = m$, it becomes energetically favorable for the bound fermion to escape the string, i.e., jump from the zero-mode state to one of the available free states nearby on the parabola. This gives a limiting value for the current.

conducting currents for a cosmic string, although these should more properly be called *persistent* currents[4]: as long as the external field is applied, the current grows and then remains constant even when E goes to zero. There is, however, a limit to this process, which is implied by the fact that the fermions are only effectively massless along the string, but massive outside the string's core, their mass being given by the vacuum expectation value of the Higgs field: when the Fermi level of the right and left movers exceeds this mass, the corresponding filled states acquire a nonvanishing probability to jump off the modes to a free particle state. Thus, there exists a maximum

[4] At sufficiently low temperatures certain materials undergo a phase transition to a new (superconducting) phase, characterized notably by the absence of resistance to the passage of currents. Unlike in these theories, no critical temperature is invoked in here, save for the temperature at which the condensate forms inside the string, the details of the phase transition being of secondary importance. Moreover, no gap in the excitation spectrum is present, unlike in the solid-state case, where the amount of energy required to excite the system is of the order of that to form a Cooper pair, and hence the existence of the gap. Having clarified this point, however, we will stick to the usual convention found in the literature in the rest of this article.

current which in fact solely depends on the current carrier's mass and charge, given by [3]

$$J_{\max} = \frac{qm}{2\pi} \tag{4}$$

which may be as large as 10^{20} A for GUT particles (10^{15} GeV).

One can see that this simple reasoning, based on energy conservation considerations, is somewhat underestimating the maximum current: if the string were straight, then it is clear that momentum could not be conserved in the escape process, so that Eq. (4) can at most provide an order-of-magnitude estimate. When a more complete analysis is made [14] on a string loop of radius R, it is then found that this equation should be transformed into $J^R_{\max} = qm^2R/2\pi$, which in principle represents an increase in the maximum allowed current. However, in practice the current cannot exceed the value of the mass of the carrier, $|J|^2 \lesssim m^2$, for otherwise they would find it energetically favorable to leave the vortex, and furthermore, even if massless along the string, the fermions can interact with each other, and multiple scattering of bound into free states tends to reduce the current. In fact, a complete (quantum) analysis, currently in progress, seems to indicate that no generic, model-independent conclusion can be drawn. The same applies to the related subject of the stability of the currents, a point whose understanding is crucial in the vorton excess problem [15].

Having discussed these elementary and general properties of fermion currents induced by zero modes, let us specialize the discussion to a particular model.

2. MASSIVE MODES

Various other models have been considered in the literature concerned with fermion bound states trapped in cosmic strings, in particular models in which not only the zero modes, but also massive bound states could be responsible for the existence of a current. In this section, we shall summarize the findings and the underlying assumptions for these models, which fall essentially in two categories: a coupling of zero modes, equivalent to a symmetry-breaking term between left and right movers, and the timelike solutions of the modified Dirac equation in a vortex. This latter category is the one we shall turn to in the next section, so for the time being let us investigate the former possibility.

Coupling zero modes through a charged Higgs field has been considered [11] in particular in the framework where the string-forming Higgs field is part of a higher-than-one-dimensional representation of the broken GUT group. It was then shown that, e.g., in the case of an $SU(2)$ doublet, the second component of the Higgs field, even though left with a vanishing expectation value outside the string's core, could condense and produce bosonic conductivity, either of the neutral or of the charged kind [13, 16]. In turn, this bosonic current could couple to the zero modes in such a way as to modify their corresponding currents, in a way depending critically on the charges of the fermions. In particular, if the bosonic current is neutral, there is hardly any change in the previously developed picture, whereas in the charged case, if the sum of the fermionic field charges were vanishing (as opposed to any other situation), almost no current would be left on the strings after a rather short dissipation time. It is clear that any conclusion about such current would be very much model dependent, so that the cosmological consequences that could be drawn would be rather inconclusive.

The previous discussion can be illuminated by a phenomenological coupling between the left and right movers, i.e., a term of the kind $\delta m \bar{\psi}_L \psi_R$, where in fact δm represents the expectation value of the second Higgs field involved, and is nonvanishing only on the worldsheet [17]. The chiral current $J^5_\mu = \bar{\psi}\gamma_\mu\gamma^5\psi$ then yields an anomaly proportional to δm and superconductivity, in principle, is no longer assured. However, in this case, a net charge per unit length (i.e., a timelike current) builds up, as could have been expected from Fig. 1: in the case at hand, the zero modes no longer exist as solutions of the "Dirac" equation, but instead one finds a second parabola underneath the free particle one, i.e., with a mass δm (see Fig. 2). Filling up states on this second parabola naturally generates a timelike current.

3. A SIMPLE MODEL AND THE PHASE FREQUENCY THRESHOLD

We shall now consider the simplest model leading to fermionic string conductivity, which, as we shall show, contains most of the characteristic features of whatever previously developed model. Consequently, we shall begin by considering the following simple model, where the background string is described by the ordinary string profile functions $\varphi(r)$ and $B(r)$, i.e., a scalar Higgs field Φ and its associated gauge vector B_μ coupled (through the gauge covariant derivatives below) with the constant q and breaking a $U(1)$ symmetry

$$\mathcal{L}_{\rm H} = \frac{1}{2}[D^{(\phi)\mu}\Phi][D^{(\phi)}_\mu\Phi]^* - \frac{1}{4}(\partial_{[\mu}B_{\nu]})^2 - \frac{\lambda}{8}(|\Phi|^2 - \eta^2)^2 \qquad (5)$$

To this background we add a set of chiral fermions Ψ whose dynamics is controlled by the Lagrangian functional

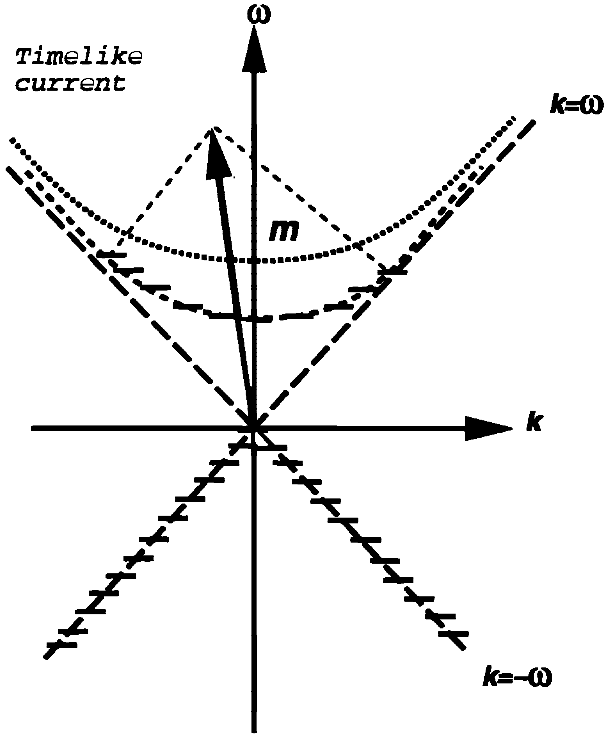

Fig. 2. Phase diagram for all modes. The zero modes are still shown as straight lines, while an extra massive mode has been included as a second parabola below the free state one, and on which states can also be filled (even though the vacuum is defined in the same manner as in the previous case involving only zero modes). The same construction as in Fig. 1 reveals that in this case timelike as well as spacelike currents can be constructed simply.

$$\mathcal{L}_{\mathrm{f}} = i\bar{\Psi}_L\gamma^\mu\mathcal{D}_\mu^{(L)}\Psi_L + i\bar{\Psi}_R\gamma^\mu\mathcal{D}_\mu^{(R)}\Psi_R - g(\bar{\Psi}_L\Psi_R\Phi + \bar{\Psi}_R\Psi_L\Phi^*) \tag{6}$$

In the case of unit winding number [$n = 1$ in Eq. (1)], it may be shown that the spinor solution takes the form

$$\Psi = e^{-i(kz-\omega t)}\begin{pmatrix} \xi_1 e^{-im\theta} \\ \xi_2 e^{-i(m-1)\theta} \\ \xi_3 e^{-i(m-1)\theta} \\ \xi_4 e^{-i(m-2)\theta} \end{pmatrix} \tag{7}$$

with $\xi_i = \xi_i(r)$ satisfying the following set of equations:

$$\frac{dX}{dr} = A(r)X \qquad \text{with} \quad X = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{pmatrix} \tag{8}$$

and

$$A(r) = \begin{pmatrix} f_R^u(r) & -i(k+\omega) & 0 & ig\varphi(r) \\ i(k-\omega) & f_R^d(r) & ig\varphi(r) & 0 \\ 0 & -ig\varphi(r) & f_L^u(r) & -i(k-\omega) \\ -ig\varphi(r) & 0 & i(k+\omega) & f_L^d(r) \end{pmatrix} \tag{9}$$

where the functions f depend only on the profile functions and vanish for $r \to \infty$. It then turns out [10] that only the angular momentum terms with $m = 1$ might have normalizable solutions. The matrix A can be split into a constant matrix plus a radius-varying one. The constant matrix has two doubly degenerate eigenvalues $\pm\sqrt{k^2 - \omega^2}$ with which one reduces the system (8) to that originally seen in the case of zero modes, with a few differences: in the zero-mode case, one finds that there are three normalizable states near the origin, whereas in the $\omega^2 \neq k^2$ case, only two are found. As we shall see below, there are also two asymptotically normalizable solutions in both cases. With three normalizable solutions near the origin and two for large r, it is always possible to match the functions in such a way that one can find an everywhere normalizable solution. With only two near the origin, another parameter must necessarily be fixed for the functions to match, and this one turns out to be energy, or more generaly $\omega^2 - k^2$. This explains why the bound states are quantized in the string, in a way reminiscent of that for particles in potentials in ordinary quantum mechanics. Figure 2 shows the new features of this simple model.

The dynamics of such a vortex will then be described by the interactions of the various states filled: they have the possibility to arrange themselves in order to relax toward an equilibrium configuration minimizing the energy and conserving the number of states below the mass threshold $m = g\eta$. States above this limit will be scattered off the string by the interactions of the type shown in Fig. 3.

That diffusion of massive bound states to free ones is possible can also be seen, surprisingly enough, at the classical level. Indeed, when one computes the asymptotic form of Eq. (8), noting the Higgs field there goes to its vacuum expectation value $\langle\Phi\rangle = \eta$, the constant part of the matrix A can then take more terms into account, and it assumes the following structure:

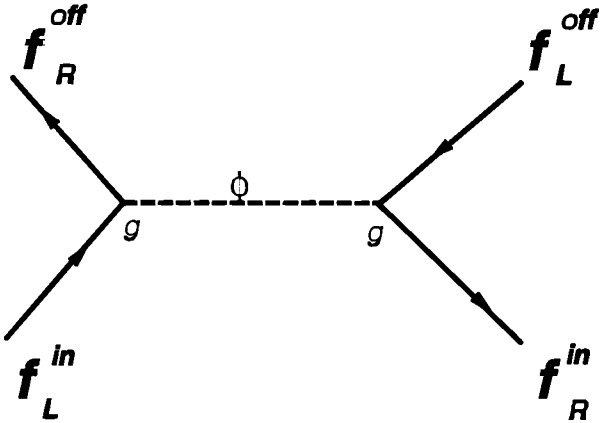

Fig. 3. Fermion interactions in the string. Those proceed via the exchange of a Higgs particle, with coupling g. The states $|f_{L,R}^{\text{in,off}}\rangle$ stand for bound ("in") incoming and free ("off") outgoing states, assuming the momenta of the incoming states are high enough so that phase space is available.

$$A|_{\text{const}} = \begin{pmatrix} 0 & -i(k+\omega) & 0 & ig\eta \\ i(k-\omega) & 0 & ig\eta & 0 \\ 0 & -ig\eta & 0 & -i(k-\omega) \\ -ig\eta & 0 & i(k+\omega) & 0 \end{pmatrix} \tag{10}$$

with eigenvalues now given by $\pm\sqrt{g^2\eta^2 + k^2 - \omega^2}$, again doubly degenerate. Since the vacuum fermion mass is precisely $m_\psi = g\eta$, one then notes the existence, exactly as in the bosonic case, of a *phase frequency threshold* above which there is no normalizable solution in the sense that, for $\omega^2 - k^2 > m_\psi^2$, the asymptotic behavior is oscillatory instead of exponentially damped, representing a superposition of incoming and outgoing cylindrical waves, i.e., free states with momentum perpendicular to the worldsheet. This was to be expected, given the analysis in terms of bound states exactly as in the ordinary quantum mechanical potential situation, and is shown clearly in Fig. 4.

As this behavior is qualitatively similar to that occurring in the case of bosonic superconductivity, it is tempting, if not yet justified, to conjecture that this might also translate into the equation of state relating the energy per unit length and the tension, and therefore that for the state parameter $\omega^2 - k^2$ tending to m_ψ^2, the tension will eventually vanish. More work is needed, however, to render this conjecture precise, and perhaps proved, in particular since the actual state parameter ought to be defined as a function of the various states filled: in principle, one should decompose the fermion field in terms of creation and annihilation operators and then write it in the

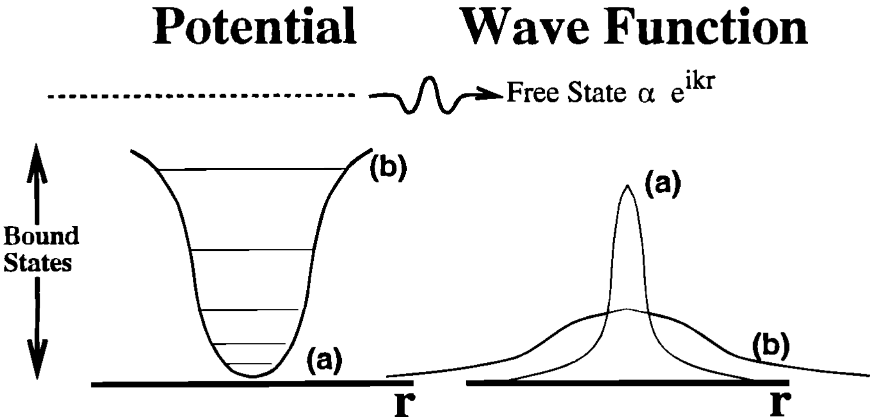

Fig. 4. Bound states in the string's background: the profile functions act on the Dirac field as a potential, and thus exhibit a quantized number of bound states, until the energy of the state exceeds the particle's vacuum mass, in which case only wave solutions, i.e., free states, can be found.

string in terms of a varying phase φ [not to be confused with the profile function $\varphi(r)$] using the equivalence (true only in the two-dimensional worldsheet)

$$\Psi\gamma^a\Psi = \frac{1}{\sqrt{\pi}}\,\varepsilon^{ab}\partial_b\varphi \tag{11}$$

where ε is the 2-dimensional Levi-Civita tensor and the numerical factor is conventional [3]. Here, the indices take values only along the worldsheet, and the actual state parameter would then be

$$w = h^{ab}\partial_a\varphi\partial_b\varphi \tag{12}$$

with h the induced metric.

4. CONCLUSIONS

In this work, we have shown that fermionic superconductivity, which we have summarized by means of an exhaustive survey of the existing literature, shares some of the same features as the bosonic one. In particular, we have investigated (and conjectured) the possibility that a phase frequency threshold might exist.

Regarding future directions, many studies still need to be pursued. Here we just comment on some of them: in the foregoing discussion we limited ourselves to the simplest vortex background case. However, one can also envisage networks of cosmic strings with winding number higher than $n =$

1. Even if from general arguments one would expect the evolving interacting network to break down into $n = 1$ strings (as is found numerically), one could be interested as well in studying fermion zero-mode solutions in a general n-vortex background (in this case there would exist n different solutions of the system [10]). In addition, one would also like to be able to describe the dynamics of the vortex with fermionic currents by means of an effective equation of state, in a way analogous to the way we describe a bosonic string [18, 19]. For that it will be necessary to compute integrated (over the vortex cross section) quantities, and hence we need to know the full behavior of the relevant fields (as encoded in the field's profile functions). This we can only attain after a full numerical solution of the system is at hand, work that is not yet done. The formalism [20] will then provide us with the necessary macroscopic description once we know the details of the internal structure of the fields. One can then think of applying these new results to the vorton problem, as it was done [21, 22] in the bosonic case.

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REFERENCES

[1] T. W. B. Kibble, *J. Phys. A* **9**, 1387 (1976); *Phys. Rep.* **67**, 183 (1980).
[2] E. P. S. Shellard and A. Vilenkin, *Cosmic Strings and Other Topological Defects*, Cambridge University Press, Cambridge (1994).
[3] E. Witten, *Nucl. Phys. B* **249**, 557 (1985).
[4] R. Jeannerot, *Phys. Rev. D* **53**, 5426 (1996).
[5] A. J. Gill and T. W. B. Kibble, *Phys. Rev. D* **50**, 3660 (1994); S. Bonazzola and P. Peter, Astropart. Phys. **7**, 161 (1997).
[6] J. Garriga and P. Peter, *Class. Quantum Grav.* **11**, 1743 (1994).
[7] A. E. Everett, *Phys. Rev. Lett.* **61**, 1807 (1988); M. Alford, K. Benson, S. Coleman, and J. March-Russell, *Nucl. Phys. B* **349**, 414 (1991); T. W. B. Kibble, G. Lozano, and A. J. Yates, *Phys. Rev. D* **56**, 1204 (1997).
[8] R. H. Brandenberger, In *Proceedings VII Mexican School of Particles and Fields and the I Latin American Symposium on High Energy Physics, Merida, Mexico, 10/29-11/6 1996* (1997) [hep-ph/9702217].
[9] A. Riotto, hep-ph/9708304.
[10] R. Jackiw and P. Rossi, *Nucl. Phys. B* **190**, 681 (1981); E. Weinberg, *Phys. Rev. D* **24**, 2669 (1981).
[11] M. Hindmarsh, *Phys. Lett. B* **200**, 429 (1988).
[12] N. K. Nielsen, *Nucl. Phys. B* **167**, 249 (1980); N. K. Nielsen and P. Olesen, *Nucl. Phys. B* **291**, 829 (1987).

[13] P. Peter, *Phys. Rev. D* **45**, 1091 (1992).
[14] S. M. Barr and A. M. Matheson, *Phys. Lett. B* **198**, 146 (1987).
[15] R. L. Davis and E. P. S. Shellard, *Phys. Rev. D* **38**, 4722 (1988); *Nucl. Phys. B* **323**, 209 (1989); B. Carter, *Ann. N.Y. Acad. Sci.* **647**, 758 (1991); R. Brandenberger, B. Carter, A. C. Davis, and M. Trodden, *Phys. Rev. D* **54**, 6059 (1996); A. C. Davis and W. B. Perkins, *Phys. Lett. B* **393**, 46 (1997).
[16] P. Peter, *Phys. Rev. D* **46**, 3335 (1992).
[17] R. L. Davis, *Phys. Rev. D* **36**, 2267 (1987).
[18] B. Carter and P. Peter, *Phys. Rev. D* **52**, R1744 (1995).
[19] B. Carter, P. Peter, and A. Gangui, *Phys. Rev. D* **55**, 4647 (1997).
[20] B. Carter, Brane dynamics for treatment of cosmic strings and vortons, in *Proceedings of the Second Mexican School on Gravitation and Mathematical Physics*, A. Garcia *et al.*, eds. Science Network Publishing (1997) [hep-th/9705172].
[21] A. Gangui and E. Gunzig, In *Proceedings of the Eighth Marcel Grossmann Meeting on General Relativity, Gravitation and Relativistic Field Theories*, T. Piran *et al.*, eds. (1998).
[22] A. Gangui, P. Peter, and C. Boehm, *Phys. Rev. D* (1998), to appear.